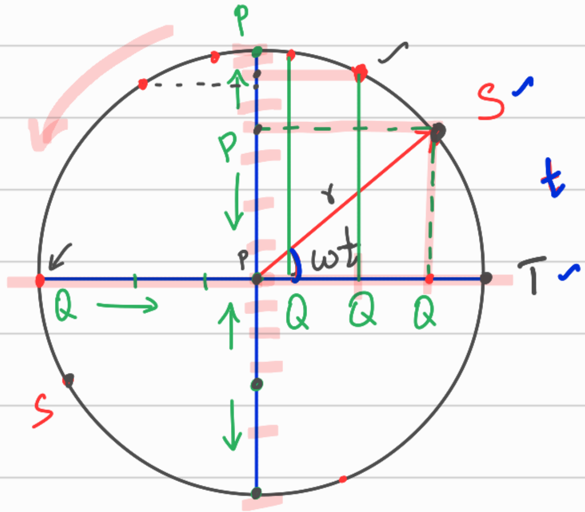
$\omega \rightarrow$ Angular Speed

Oscillatory motion



$$V = \frac{S}{t}$$

$$\boxed{\omega = \frac{d\theta}{dt}}$$

$\underline{d\theta = \omega dt}$

$\underline{\theta = \omega t}$

* Force acting towards mean position
* Force directly prop. to displacement
* Harmonic function sin θ & cos θ

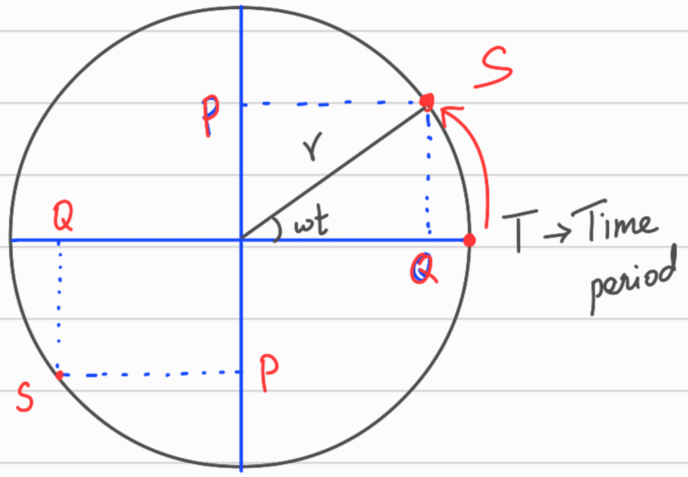
θ	$\sin\theta$	$\cos\theta$
0°	0	1
90°	1	0
180°	0	-1
270°	-1	0
360°	0	1
450°	1	0

$$y = a\sin\theta$$

$$\omega = 2\pi f$$

540°	0	-1
720°	-1	0

↳ Point S is making uniform circular motion



↳ Angular speed ω

↳ Time period - T

↳ ν (nu) → frequency → No. of cycles per unit time

$$\nu = \frac{1}{T}$$

$$f = \frac{1}{T}$$

T → 2π

1s → $\frac{2\pi}{T} = \omega$

2π → Ang. disp

T → Time

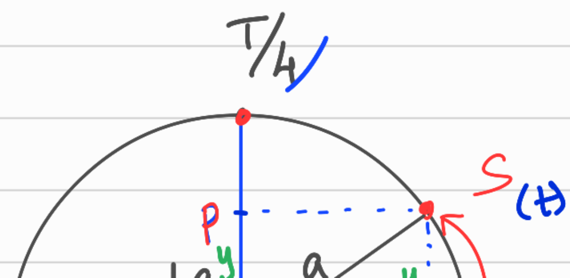
$\omega = \frac{\text{Avg disp}}{\text{Avg ti}}$

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$$

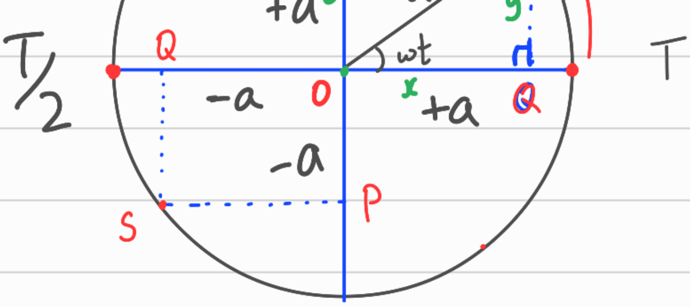
$\omega = 2\pi f$

Displacement equation

OP is displacement 'y'



at time 't', maximum displacement is $\pm a$



$$\frac{y}{a} = \sin\omega t \quad \text{or} \quad \boxed{y = a\sin\omega t}$$

Equation of displacement when $t = 0, y = 0$

$t = T/4$

$$y = a\sin\left(\frac{\cancel{2}\pi}{\cancel{T}}\right)\frac{\cancel{T}}{\cancel{4}_{2}}$$

$$y = a\sin\left(\pi/2\right) \qquad \boxed{y = a}$$

when $t = T/2$, $y = a\sin\left(\frac{\cancel{2}\pi}{\cancel{T}}\right) \times \frac{\cancel{T}}{\cancel{2}}$

$$y = a\sin\pi$$

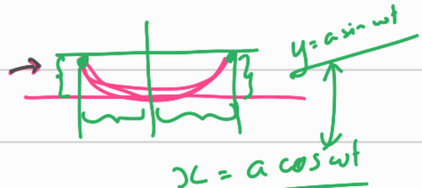
$$y = 0$$

when $t = \frac{3T}{4}$

$$y = a\sin 270^\circ$$

$$= a \times (-1) = -a$$

$$y = -a$$



$$\frac{x}{a} = \cos \omega t \rightarrow \boxed{x = a \cos \omega t}$$

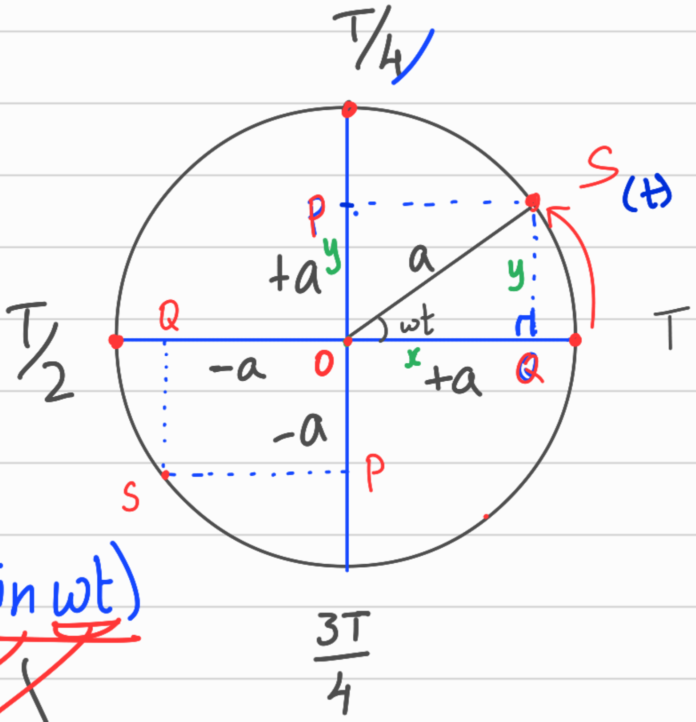
$t = 0 \rightarrow x = a$ starting point

$t = \frac{T}{4} \rightarrow x = 0$ $t = \frac{T}{2}$ $x = -a$

$t = \frac{3T}{4} \rightarrow x = 0$

Velocity equations

Displacement



$$\boxed{y = a \sin \omega t}$$

Velocity $V = \frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t)$

$$= a \cos \omega t \cdot \omega$$

$$\frac{dy}{dt} = a \frac{d}{dt} \sin \omega t \times \frac{d}{dt}(\omega t)$$

Relate V & t → $\boxed{V_t = a \omega \cos \omega t}$

$$V_t = \omega \sqrt{a^2 \cos^2 \omega t}$$

$$= \omega \sqrt{a^2 (1 - \sin^2 \omega t)}$$

$$= \omega \sqrt{a^2 - a^2 \sin^2 \omega t}$$

f 'y'

$0 = \omega \sqrt{a^2 - y^2}$

V & displa...

$$\boxed{V = \omega\sqrt{a^2 - y^2}}$$

$0 = a^2 - y^2$

Minimum velocity

$V = 0$ when $y = \pm a$ at extreme

Maximum Velocity

$V = V_{max}$ $\Rightarrow$ $y = 0$ at mean position

$$V_{max} = \omega\sqrt{a^2 - 0} = \omega a$$

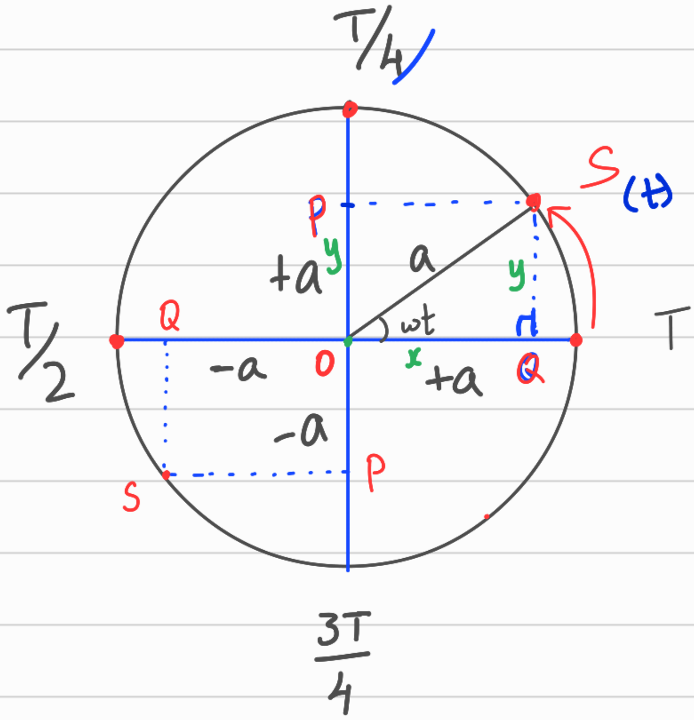
$$V_{max} = a\omega$$

Acceleration :

displacement equation

$$\boxed{y = a\sin\omega t}$$

$$\left.\begin{aligned} V &= a\omega\cos\omega t \\ V &= \omega\sqrt{a^2 - y^2} \end{aligned}\right\}$$



Acceleration,

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}\, a\omega\cos\omega t$$

dt dt

$$= a\omega(-\sin\omega t \cdot \omega)$$

$$= -a\omega^2 \sin\omega t$$

$$\boxed{\alpha = -\omega^2 y}$$

Special Cases:

Minimum acceleration when $y = 0$ at mean position $a = 0$

Maximum α at $y = \pm a$ at the extremes

$$\alpha_{max} = \pm\omega^2 a$$

$$\alpha = -\omega^2 y$$

↳ Negative sign shows that direction of y & a are opposite to one other.

① → Acceleration is directly prop. to displacement

② $\alpha = -\omega^2 y$

Neg. sign shows that direction of disp & acceleration are opposite to each other

↳ Simple Harmonic Motion (SHM)

$$\omega^2 y = |\alpha|$$

$$\omega = \sqrt{\frac{\alpha}{y}} \qquad \frac{2\pi}{T} = \sqrt{\frac{\alpha}{y}}$$

$$T = 2\pi\sqrt{\frac{y}{\alpha}}$$

$$\boxed{T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}}$$

Problem

A body oscillates with SHM according to the equation,

$$x = 5_{(m)}\left[\cos 2\pi_{(rad/s)}\, t + \pi/4\right]$$

At $t = 1.5s$, calculate (a) displacement (b) speed and (c) acceleration of the body.

a) Displacement

$$\frac{1}{\sqrt{2}} = \frac{1}{1.414}$$

$$\frac{1}{\sqrt{2}} = 0.707$$

$$x = 5\cos(2\pi t + \pi/4)$$

$$x = 5\cos(2\pi(1.5) + \pi/4)$$

$$= 5\cos(3\pi + \pi/4)$$

$$= -5\cos\pi/4 = -5 \times 0.707$$

$$= -\underline{3.535}\ m.$$

b) Velocity

no 't'

$$\frac{d\left(\frac{\pi}{4}\right)}{dt} = 0$$

$$V = \frac{dx}{dt} = \frac{d}{dt}\left(5\cos(\underline{2\pi t + \pi/4})\right)$$

$$= -5\sin(2\pi t + \pi/4) \times 2\pi$$

$$= -5 \times 2\pi \sin(2\pi \times 1.5 + \pi/4)$$

$$= +5 \times 2\pi \sin\pi/4 = \underbrace{5 \times 2}_{} \times \frac{22}{\cancel{7}} \times \cancel{0.707}\ 0.101$$

$$= 22.22\ m/s$$

$$= 220 \times 0.101$$

c) Acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(-10\pi\sin(2\pi t + \pi/4)\right)$$

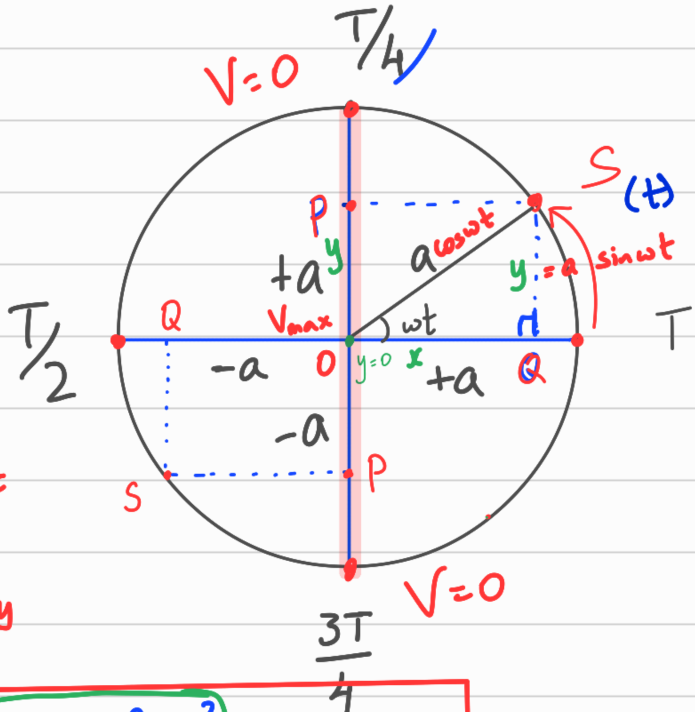
$$= -40\ldots$$

$= 140\ m/s$

Energy :-

K.E at displacement y

Let the mass be 'm'



$$V = \pm a\omega \cos\omega t \rightarrow V_t$$

$$V = \omega\sqrt{a^2 - y^2} \rightarrow V_y$$

$$K.E = \frac{1}{2}mv^2 \qquad \boxed{K.E = \frac{1}{2}ma^2\omega^2\cos^2\omega t}$$

$$KE = \frac{1}{2}m\omega^2(a^2 - y^2)$$

Max. $K.E = \frac{1}{2}\underline{m\omega^2 a^2}$ at mean position

Min. $K.E = 0$ at extremes

Potential :-

$$\text{Force} = m\alpha = -my\omega^2$$

$$\text{Workdone} = \int_0^y F\,dy = \int_0^y -my\omega^2\,dy$$

$$= \left[m\omega^2\frac{y^2}{2}\right]_0^y = \frac{1}{2}m\omega^2 y^2$$

$$= \frac{1}{2}m\omega^2 a^2 + 0$$

$$\frac{1}{2}m\omega^2 a^2$$

$= 0 + \frac{\ }{2}$

$$\boxed{P.E. = \frac{1}{2} m\omega^2 y^2}$$

$\frac{1}{2} m\omega^2 a^2 (\cos \omega \ldots \sin^2 \omega t)$

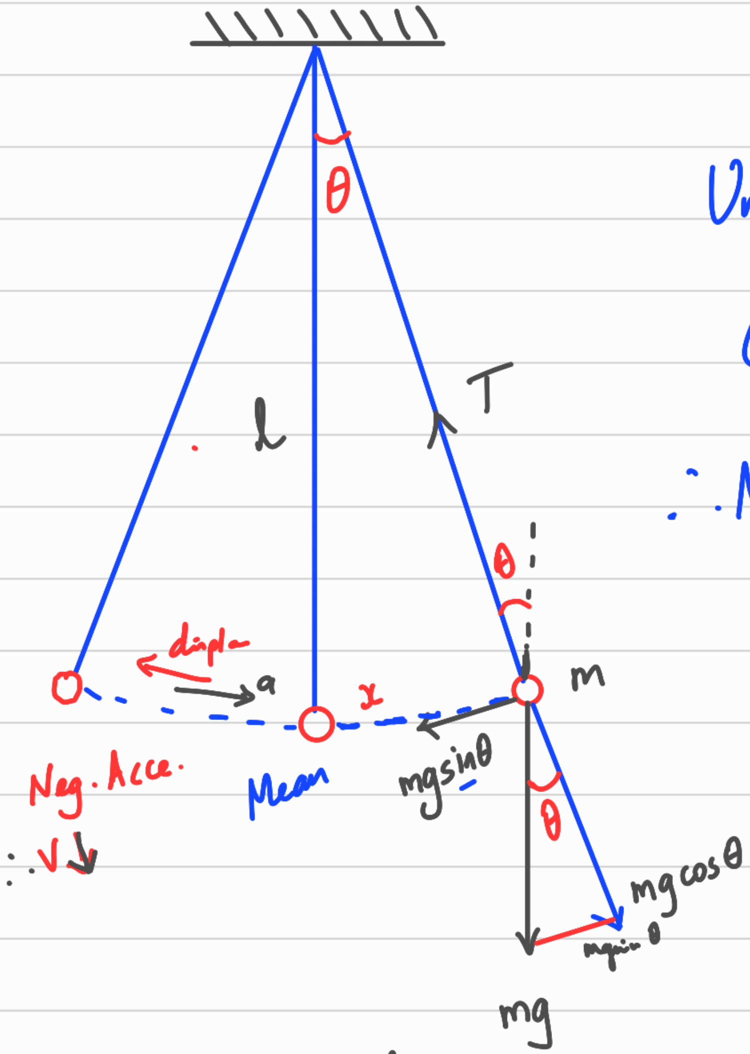
$$\boxed{PE = \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t}$$

$= \frac{1}{2} m\omega^2 a^2$

✓ Minimum $PE = 0$ when $y = 0$ (at mean position)

✓ Maximum $PE_{max} = \frac{1}{2} m\omega^2 a^2$ when $y = a$

Simple Pendulum:



$T = mg\cos\theta$

Unbalanced force → $mg\sin\theta$

Create acceleration → a

∴ Net force

$\cancel{m}a = \cancel{M}g\sin\theta$

$a = g\sin\theta$

For smaller value of

$\theta \rightarrow \sin\theta = \theta = \frac{x}{l}$

$\frac{x}{a} = \frac{l}{g} \longleftarrow a = g\frac{x}{l}$

A pendulum is displaced through an angle of θ from its mean position

(1) $a \propto x$ (SHM) Simple Harmonic Motion

acceleration $\propto$ displacement

(2) A & D opposite in direction

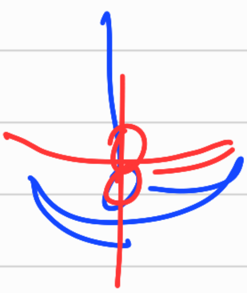
$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{x}{a}}$$

$$\boxed{T = 2\pi\sqrt{l/g}} \qquad T \propto \sqrt{l}$$

Prob:

What is the length of a simple pendulum, which ticks seconds?

$T = 2s$ $\quad g = 9.8\,m/s^2$



$l = 0.99\,m$